Optimized Algorithm for Probabilistic Evaluation of Enhanced Distributed Coordination Access According to IEEE 802.11e

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***Abstract*—The IEEE 802.11e standard introduces a system of Access Categories (AC) and differentiated medium access control mechanisms for wireless local area networks. The preferential or deferral treatment of frames is achieved using configurable Arbitration Inter-Frame Spaces (AIFS) and customizable Contention Window (CW) sizes. In this paper we introduce a general algorithm which calculates the probability of winning the contention by a given AC, based on the values of AIFS lengths and CW sizes.**

I. Introduction

During the last several years the quality of service control has become an essential part of modern wireless network technologies. Standards and drafts such as IEEE 802.11e or 802.11n introduce differentiated packet treatment for wireless local area networks (WLAN), but the expansion of the usage of these features is conditional on a simple projection method of real QoS requirements into corresponding configuration parameters. The aim of our work is to derive mathematical relation between the probability of winning access to a wireless medium and configuration parameters such as Arbitration InterFrame Space (AIFS) length and Contention Window (CW) size. For this purpose we use probabilistic calculus.

In paper [6] we introduced a procedure to derive mathematical formulas expressing the degree of mutual prioritization between two access categories in relation to the interframe spaces and contention window sizes assigned to them. In this paper we will present a generalized method with arbitrary finite number of WLAN stations, as introduced in [7], and extended by the optimization of the computational efficiency.

Section II compares the classical 802.11a/b/g MAC methods with those available in the IEEE 802.11e standard. It also identifies the major configurable MAC parameters which affect the access priority. Section III then describes a model which was the subject of our mathematical analysis. Algorithms for computing the desired probabilities are presented. In Section IV we connect the model with the real situation parameters and provide two examples with results. Finally, the results are interpreted.

II. Medium Access Control functions in IEEE 802.11

A. MAC functions in 802.11 a/b/g

Majority of recent WLANs use only the fully distributed, contention-based Medium Access Control mechanism called Distributed Coordination Function (DCF). According to DCF, the station having data to send must win a contention with other stations to gain access to the shared radio channel. The contention is based on the combination of time constants and random waiting periods.

Time constants represent the minimum waiting periods between two frames immediately following each other. These constants are called InterFrame Spaces (IFS). We distinguish three types of them:

- Short InterFrame Space (SIFS), as the name suggests, represents the shortest waiting period, which corresponds with the respective highest priority level.
- If DCF operates in combination with centralized polling called Point Coordination Function (PCF), the access point has to wait for the PCF InterFrame Space (PIFS) to start polling the registered stations. PIFS ensures midlevel priority.
- Most often stations transmit frames containing user data. In this case the stations must wait for DCF InterFrame Space (DIFS) before they can initiate data transmission.

To avoid simultaneous access from all the competitors after DIFS has expired, each station has to wait for an additional random period. This random waiting time is generated from the range of 0 to CW, labelled according to the name Contention Window. The random number generated by each station is decreased by the end of each "Slot time". When zero is achieved (if ever), the station can access the medium. After the winning station has started transmitting, all the other stations should detect that the medium is occupied, stop their countdown and save the recent value for the next competition.

Sometimes happens that two or more stations choose the same random value, which means that they would gain access to the shared medium and start to transmit data at the same

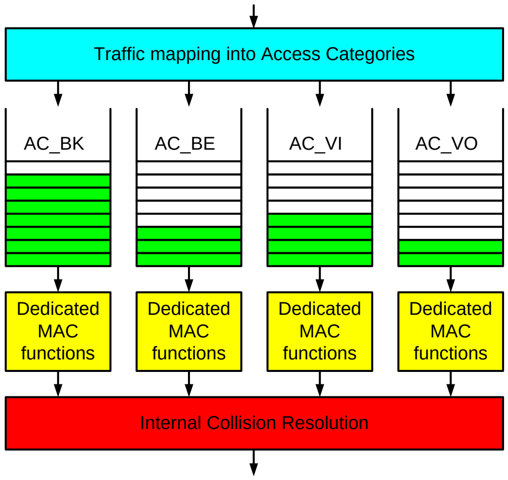

Fig. 1. IEEE 802.11e functional model

time. Such a situation is called collision. In such a case the contention window is increased in order to reduce the probability of the selection of the same random number during the next attempt.

According to [4] the corresponding time intervals for IEEE 802.11g are the following: $SIFS = 10\,\mu$s, $DIFS = 50\,\mu$s, and $Slot_time = 20\,\mu$s.

B. MAC functions in 802.11e

The aim of the IEEE 802.11e standard is to overcome the limitations of the original 802.11 a/b/g/ MAC algorithm by allowing traffic flows to be classified into several service classes, and to offer differentiated treatment for these classes. The differentiation in IEEE 802.11e is achieved by assigning a customizable set of MAC parameters to the Access Categories (AC). There are four access categories defined in the 802.11e standard [5]:

- AC_VO for real-time, voice-based, conversational services,
- AC_VI for video services,
- AC_BE for standard best-effort services, covering the majority of network applications,
- AC_BK for background services for which a priority lower than the one assigned to the standard network applications is sufficient.

As shown in Fig. 1, the outgoing traffic is classified into the four available traffic classes. To offer differentiated packet treatment, a configurable set of parameters is assigned to each AC. The parameters which directly control the MAC processes are:

- $AIFS[AC]$ – represents the initial interframe space (Arbitration InterFrame Space) for the given AC,
- $CW_{\min}[AC]$ – represents the initial size of the contention window for the given AC,
- $CW_{\max}[AC]$ – represents the maximum size of the contention window for the given AC,
- $AF[AC]$ – represents the increase factor of the contention window when collision occurs during the transmission; the new contention window is calculated according to the formula $CW_{\min}^{\text{new}}[AC] = (CW_{\min}^{\text{old}}[AC]+1)\cdot AF[AC]-1$.

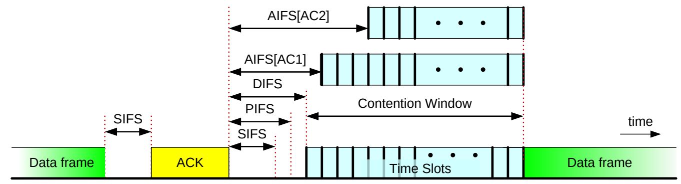

Fig. 2. Contention process between different access categories

The contention-based distributed MAC function in 802.11e is called Enhanced Distributed Coordination Access (EDCA). In spite of the constant $DIFS$ in DCA, EDCA introduces different $AIFS$s and contention windows for the available access categories. Thus if an access category must be prioritized, it can be realized by either being assigned a shorter $AIFS$ or reducing the contention window of the AC or by the combination of the two approaches. Fig. 2 shows the contention process between access categories of different priorities.

The 802.11e standard specifies only the set of configurable parameters for the access categories but does not define the mutual relation between these values. In the next Section we analyze mathematically the behavior of a simplified model of 802.11e EDCA and derive the relation between the values of $AIFS$ and CW and the corresponding degree of prioritization of the access category.

III. Probabilistic Analysis of Simplified Model of IEEE 802.11e

The aim of our mathematical analysis is to express mathematically the degree of mutual prioritization between several access categories in relation to the interframe spaces and contention windows assigned to these categories. We assume that the situation we model is not determined by the result of the preceding contention. In such a case we can assume that all stations generate a random value from their default contention window only. This simplification, which more or less correspond with lightly loaded WLAN network, leads to a more transparent mathematical derivation but, on the other hand, it constrains the relevance of the final results.

A. Notation and Goal of Analysis

A discrete random variable X with a discrete uniform distribution on the set of integers $\{a, a+1, \ldots, b\}$, $a \leq b$, will be denoted $X \sim \mathrm{Ud}(\{a, \ldots, b\})$.

In our model, we deal with a total number of K stations, each of them having its $AIFS$ and $CW_{\min}$. The initial interframe space for the k-th station, $k = 1, \ldots, K$ will be denoted by N_k^0, its contention window size by N_k and their sum by $\Sigma_k = N_k^0 + N_k$. To connect the mathematical notation and the theory (proved in detail in Section IV):

$$AIFS[ACk] = N_k^0, \quad CW_{\min}[ACk] = N_k - 1. \tag{1}$$

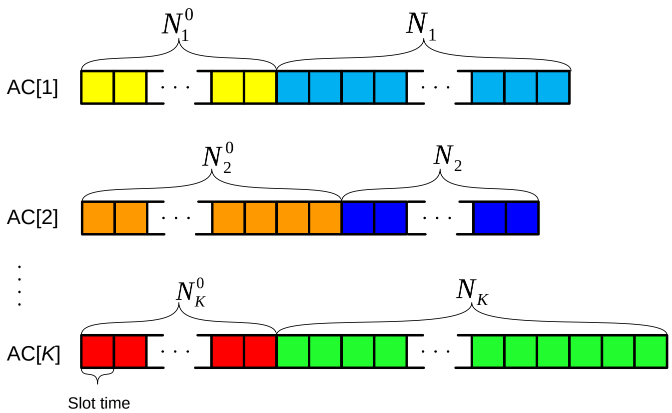

Fig. 3. Model of K access categories with different $AIFS$s and CWs

In the contest, each station can be considered a generator of a random integer. We will model the respective stations as random variables

$$X_k \sim \mathrm{Ud}(\{N_k^0+1,\ldots,N_k^0+N_k\}), \quad k=1,\ldots,K. \qquad (2)$$

The variables $X_1,\ldots,X_K$ are statistically independent. The mathematical model of the situation is illustrated in Fig. 3.

The goal of our analysis is to find the probability P_{win}^1 that the first competitor (AC1) wins. Hereat, the choice of the "first" one is with no loss of generality, so in the following text we will mostly use just P_{win}. In case we will be interested in such probability for an other station, we will use symbol P_{win}^k. Secondly, we want to compute the probability P_{coll} that a collision occurs.

Specifically, the first station X_1 wins the competition in the case that the random number generated by the station will be smaller than any of the numbers generated by the other stations $X_2,\ldots,X_K$. The collision means that there are at least two identical numbers generated, which are smaller than all the others (if any) at the same time.

B. Derivation and Results in Case of Two Stations

In this Section, the number of the access categories considered is limited to two. We have AC1 and AC2, with their respective interframe spaces N_1^0 and N_2^0, and with their respective contention windows of sizes N_1 and N_2. And, of course, the independent random variables X_1 and X_2. The goal is to find the probabilities $P_{\text{win}}=P(X_1<X_2)$ and $P_{\text{coll}}=P(X_1=X_2)$ in terms of parameters N_1^0,N_1,N_2^0,N_2.

Such a problem is completely solved in [6]. It is shown that even for as little as two stations, general formulas for P_{win} and P_{coll} cannot be found. In fact, we have to distinguish several cases, taking into account relations between the parameters. The six cases denoted by the letters A to F are presented in Table I. Because the two stations do nothing during the common part of the interframe periods N_1^0 and N_2^0, we simplify the situation by introducing the difference term $d=N_1^0-N_2^0$.

C. Derivation and Results for General Number of Stations

As in case $K=2$ a general formula does not exist, it is naturally not possible to derive such a formula in the more complicated case of $K\geq 2$. Thus, the computation of the desired probabilities will have to be of algorithmical character. (However, explicit formulas can be derived for special cases like all the stations having same $AIFS$s and $CW_{\min}$s.)

1) The first station's probability of winning: We have to evaluate the probability

$$P_{\text{win}} = P(X_1<X_2 \wedge X_1<X_3 \wedge \ldots \wedge X_1<X_K). \qquad (3)$$

The random events $(X_1<X_k)$ for $k=2,\ldots,K$ are however not mutually independent. We will decompose the complex event stated in (3) into a combination of independent events

$$\begin{aligned}&[(X_1=N_1^0+1)\wedge(X_2>N_1^0+1)\wedge\ldots\wedge(X_K>N_1^0+1)]\\ &\vee\\ &[(X_1=N_1^0+2)\wedge(X_2>N_1^0+2)\wedge\ldots\wedge(X_K>N_1^0+2)]\\ &\vee\cdots\vee \qquad (4)\\ &[(X_1=\Sigma_1)\wedge(X_2>\Sigma_1)\wedge\ldots\wedge(X_K>\Sigma_1)],\end{aligned}$$

so that now, by the classical rules of probability theory, $\vee$ could be replaced by summation and $\wedge$ by multiplication. This simplifies (3) to

$$\begin{aligned}P_{\text{win}} = \;& P(X_1=N_1^0+1)\cdot\prod_{k=2}^{K}P(X_k>N_1^0+1)\\ &+P(X_1=N_1^0+2)\cdot\prod_{k=2}^{K}P(X_k>N_1^0+2)\\ &+\ldots \qquad (5)\\ &+P(X_1=\Sigma_1)\cdot\prod_{k=2}^{K}P(X_k>\Sigma_1).\end{aligned}$$

Because X_1 attains any of the values from $\{N_1^0+1,\ldots,\Sigma_1\}$ with probability $\frac{1}{N_1}$, equation (5) could be rewritten as

$$P_{\text{win}} = \frac{1}{N_1}\sum_{i=1}^{N_1}\left[\prod_{k=2}^{K}P(X_k>N_1^0+i)\right]. \qquad (6)$$

For particular fixed k and i, the term $P(X_k>N_1^0+i)$ is related to the number of integers belonging to the k-th contention window $\{N_k^0+1,\ldots,\Sigma_k\}$, being greater than N_1^0+i at the same time. See, for example, the illustration in Fig. 3. Because the probability is distributed uniformly over these possible occurrences, the resulting number has to be divided by the number of integers able to be chosen, i.e. by N_k. So the previous equation could be technically rewritten as

$$P_{\text{win}} = \qquad (7)$$
$$\frac{1}{\prod_{k=1}^{K}N_k}\sum_{i=1}^{N_1}\prod_{k=2}^{K}\left|\{a|a>N_k^0\wedge a\leq\Sigma_k\wedge a>N_1^0+i\}\right|,$$

where $|M|$ denotes the cardinality of set M.

TABLE I
TABLE SUMMARIZING RESULTS IN CASE OF TWO STATIONS

	$d \le 0$			$d > 0$		
	$N_1 < 1-d$	$N_1 \ge 1-d$		$N_2 > d$		$N_2 \le d$
		$N_1 \le N_2 - d$	$N_1 > N_2 - d$	$N_1 \ge N_2 - d$	$N_1 < N_2 - d$	
P_{win}	1	$1 - \frac{(N_1+d)(N_1+d+1)}{2N_2N_1}$	$\frac{N_2-2d-1}{2N_1}$	$\frac{(N_2-d)(N_2-d-1)}{2N_2N_1}$	$1 - \frac{N_1+2d+1}{2N_2}$	0
P_{coll}	0	$\frac{N_1+d}{N_2N_1}$	$\frac{1}{N_1}$	$\frac{N_2-d}{N_2N_1}$	$\frac{1}{N_2}$	0
denotation	A	B	C	D	E	F
scheme						

The above result is now transformed into the algorithm for the computation of P_{win} below. The algorithm takes advantage of the possibility to store the particular results into a matrix of predefined size, from which the final result is computed by multiplication and summation (as stated in (7)). We say again that we look for the probability that the first station wins, i.e. this station is specified by N_1^0 and N_1.

Algorithm 1 (Computation of P_{win} for $K \ge 2$): Let the constants N_k^0, N_k be given for all $k = 1, \dots, K$.

1) Allocate memory for a matrix of size $K-1 \times N_1$.
2) (optional step) Find $m = \min_k\{N_k^0\}$ and replace N_k^0 by $(N_k^0 - m)$ for all k.
3) Compute $\Sigma_k = N_k^0 + N_k$ for all k.
4) For $n = (N_1^0 + 1) : \Sigma_1$ (all possible numbers generated by the first station, the outer loop):
   For $k = 2 : K$ (i.e. all the other stations, the inner loop):
   Count up all numbers a satisfying $a \in \{N_k^0+1, \dots, \Sigma_k\}$ and $a > n$. Store the result in the matrix, row $k-1$, column $n - N_1^0$.
5) After filling the matrix, multiply all its elements in each column and then sum the resulting N_1 numbers. Dividing this by $\prod_k N_k$, obtain P_{win}.

2) Probability that collision occurs: The probability could be computed using a simple trick. It is clear that the contest of the stations ends by either winning of a station or a collision. Thus, utilizing the complementary probability, we can put down

$$1 = P_{\text{win}}^1 + P_{\text{win}}^2 + \dots + P_{\text{win}}^K + P_{\text{coll}}. \tag{8}$$

Algorithm 2 (Computation of P_{coll} for $K \ge 2$):

1) For $k = 1 : K$
   Compute the probability P_{win}^k that the k-th station wins.
2) Sum up the numbers. Denote the result by s.
3) Compute P_{coll} as $1 - s$.

IV. EXAMPLES

According to the IEEE 802.11g and 802.11e standards we can define a scenario containing a mix of mobile stations some of which offer QoS support and others do not. A review of parameters used can be found in Table II.

To match the two cases, however, let us note that there is a difference between 802.11g and 802.11e at the moment when the state of the medium is evaluated. In the case of 802.11g the first evaluation of the state is executed after the lapse of the first *Slot_time*. In the case of 802.11e the first evaluation is processed immediately after the *AIFS*. In our calculations this diversity can be eliminated if we suppose that for 802.11g the interframe space equals $(DIFS + 1 \cdot Slot\_time)$. Graphical representation of the resultant situation can be found in Fig. 4.

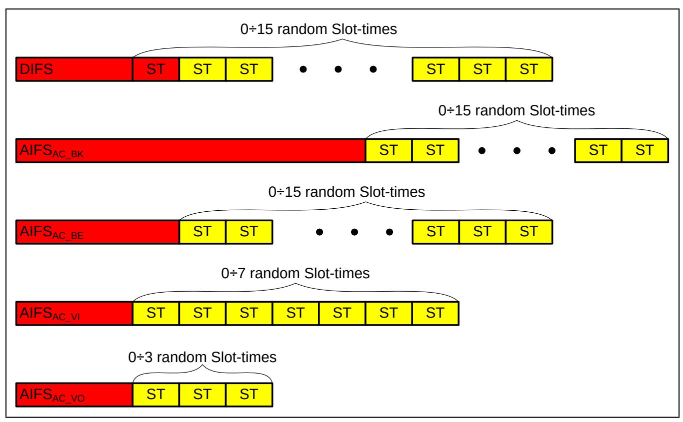

Fig. 4. Graphical representation of the MAC parameter for the demonstration scenario

In our scenarios we use the *AIFSN* and CW_{min} values specified in Table 7-37 of the IEEE standard [5]. The relation between *AIFSN* and the *IFS* length is $IFS = SIFS + AIFSN \cdot Slot\_time$.

To keep relation between the mathematical formulas and the 802.11 configuration clear we used a very basic model of

TABLE II
REVIEW OF MAC PARAMETERS

Technology	Access category	AIFSN	IFS length	CW_{min}
802.11g	—	—	50 μs (DIFS)	15
802.11e	AC_BK	7	150 μs	15
802.11e	AC_BE	3	70 μs	15
802.11e	AC_VI	2	50 μs	7
802.11e	AC_VO	2	50 μs	3

TABLE III
RESULTS FOR PARTICULAR P_{win}S AND P_{coll} IN FIRST SCENARIO

k	Type	$AIFSN$	$CW_{\min}$	$P_{\mathrm{win}}^{k} \cdot 100$
1	802.11e – AC_VI	2	7	16.03
2	802.11e – AC_VO	2	3	50.97
3	802.11e – AC_BE	3	15	2.59
4	802.11e – AC_BE	3	15	2.59
5	802.11e – AC_BK	7	15	0
6	802.11g	3	15	2.59
7	802.11g	3	15	2.59
Collision probability $P_{\mathrm{coll}} \cdot 100$				22.66

TABLE IV
RESULTS FOR PARTICULAR P_{win}S AND P_{coll} IN SECOND SCENARIO

k	Type	$AIFSN$	$CW_{\min}$	$P_{\mathrm{win}}^{k} \cdot 100$
1	802.11g	3	15	20.80
2	802.11g	3	15	20.80
3	802.11e – AC_BK	7	15	3.81
4	802.11e – AC_BE	3	15	20.80
5	802.11e – AC_BE	3	15	20.80
Collision probability $P_{\mathrm{coll}} \cdot 100$				12.99

interframe spaces N_k^0 and contention window sizes N_k. For the practical interpretation we indicate the conversion between the real network parameters and the parameters used in our formulas.

First, in the case of 802.11g the conversion of value $DIFS$ to an $AIFSN$ should be defined. According to the values presented in the example we can write $DIFS = SIFS + 2 \cdot Slot\_time$. The 802.11g station will check the state of the medium at time $DIFS + Slot\_time = SIFS + 3 \cdot Slot\_time$. It means that for 802.11g we write $AIFSN = 3$.

Based on the results of the previous paragraph we are able to express the length of the interframe space for the k-th station of the scenario as $(SIFS + AIFSN[k] \cdot Slot\_time)$ and the duration of the contention window as $(N_k \cdot Slot\_time)$. It means that for our calculation each interframe space can be shortened by $SIFS$ without any effect on the probability of winning the contention. After this shortening each of the remaining parameters is a multiple of Slot time, so we can write exactly the same as in Eq. (1).

The settings and the results of our analysis for two scenarios are given in Tables III and IV. Corresponding to the first of them, the intermediate matrix used within Algorithm 1 is presented in Table V. This matrix of size $(K-1) \times N_1 = 6 \times 8$ is generated in the case that we ask for the probability of the station No. 1 (in accordance with the analysis assumption) to win the contention, P_{win}^1. Directly following (7), after the matrix has been completed, elements within each column are multiplied, the results are summed up and the final number, 0.1603, is obtained by dividing it by $\prod_{k=1}^{K} N_k$.

V. INTERPRETATION OF RESULTS

The results of our analysis correspond to the empirical expectations. The AC_VO category has a very large probability to gain access to the medium. All best-effort (AC_BE) access

TABLE V
MATRIX USED WITHIN ALGORITHM 1 FOR CASE OF LOOKING FOR PROBABILITY OF FIRST STATION TO WIN. ONLY FIRST THREE ROWS HAVE NON-ZERO PRODUCT.

Station No.								
2	3	2	1	0	0	0	0	0
3	16	15	14	13	12	11	10	9
4	16	15	14	13	12	11	10	9
5	16	16	16	16	16	15	14	13
6	16	15	14	13	12	11	10	9
7	16	15	14	13	12	11	10	9

categories have the same probability to gain an access, but this probability is significantly smaller than that of the AC_VO. According to the selected configuration values we can also see that when there is a AC_VO frame, the background access category has no chance to win the contention. After removing the high-priority traffic the P_{win} of the best effort categories greatly increases.

We can also mention a significant change in the probability of a collision. We can conclude that high priority traffic will increase the probability of collisions. On the other hand, the decrease of the P_{coll} in Table IV was also caused by fewer competing access categories. From this we can deduce that to have a WLAN network operate efficiently the amount of high priority traffic should be kept at a sufficiently low rate.

VI. OPTIMIZATION OF ALGORITHM

Looking at the matrix that is applied in Algorithm 1 (see, for example, Table V), three regularities that build up its structure can be clearly identified:

1) first section (from left to right) of a row can contain constant values,
2) from that column onwards the values decrease to the right always by unity,
3) reaching zero this way, the third section consists of zeros until the last column.

The first and the third sections could be of zero length. All these statements are connected with formula (5) or (7) and will be discussed in this sense below. In Section VI-C the optimized algorithm will be given.

A. Discussion of regularity 3) — columns containing zeros

When the situation arises that a zero appears in some position of the matrix (numbers cannot go below zero since the number of elements in a set is concerned here), it means in particular that a column containing zero is redundant for the calculation since the product of elements in this column will always be zero and thus will not contribute to the probability sought.

It is also obvious that once a zero appears, all the columns to the right will also contain it. The number of columns in the matrix that are worth considering can be pre-calculated so that fewer calculations and less storage space are necessary. Let us write

$$\Sigma_{k^*} = \min\{\Sigma_2, \ldots, \Sigma_K\}. \quad (9)$$

1) The case $\Sigma_1 < \Sigma_{k^*}$: In this case each of the stations $k = 2, \ldots, K$ has a chance to lose the contention, no matter what the admissible value of X_1 is. This means that the expression $P(X_k > N_1^0 + i)$ is, necessarily, positive for all admissible i and thus all $\Sigma_1 - N_1^0 = N_1$ addends in equation (5) are non-zero and contribute to the result. This corresponds to the situation when there is absolutely no zero in the matrix and thus all the N_1 columns need to be used.

2) The case $\Sigma_1 \geq \Sigma_{k^*}$: This means that there is a station $k^* \in \{2, \ldots, K\}$, for which there must be i such that $P(X_{k^*} > N_1^0 + i) = 0$; this is to say that if the first station chooses $X_1 = N_1^0 + i$, there is no chance of station k^* choosing a higher number.

The least such i for such a station k^* is calculated as $(\Sigma_{k^*} - N_1^0 - 1)$. But since it may also happen that $N_1^0 > \Sigma_{k^*} - 1$ and thus such i comes out negative (which does not make sense), the number of non-zero addends in (5) and, consequently, columns in the matrix is determined as

$$n_{\text{col}} = \max\{0, \Sigma_{k^*} - N_1^0 - 1\}. \tag{10}$$

Example: The matrix from Algoritmu 1 is in Table V. The above-described qualities can be seen in the matrix: $\Sigma_1 = 2 + 8 = 10$, while $\Sigma_{k^*} = \Sigma_2 = 2 + 4 = 6$. According to (10), the number of meaningful columns equals $\Sigma_2 - N_1^0 - 1 = 3$; the other columns are redundant.

B. Discussion of regularities 2) and 1) — sequences decreasing by one

In Algorithm 1, all elements of the matrix have to be calculated; it is evident that using relatively simple rules the matrix elements could be generated more rapidly. We will try to do so throughout the following lines.

Considering possible configurations of the stations, it can be derived (see the Appendix), that for $i \leq N_k^0 - N_1^0$, $j > N_k^0 - N_1^0$, $i, j \in \{1, \ldots, \Sigma_1 - N_1^0\}$ it will hold for any (but fixed) station $k \in \{2, \ldots, K\}$:

$$1 = P(X_k > N_1^0 + i) > P(X_k > N_1^0 + j). \tag{11}$$

The maximum probability has thus been reached for the indexes $1 \leq i \leq N_k^0 - N_1^0$. The number of such indexes is thus

$$i_{\max}(k) = \max\{0, N_k^0 - N_1^0\}. \tag{12}$$

Thus if the significant columns in the matrix are only taken into consideration (see Section VI-A), it is sufficient to determine its first column, to begin with. The remaining columns are substituted via decrementing by one; in a specific row, however, reduction only takes place if $i_{\max}(k)$ columns have been calculated/filled in this way; otherwise the respective value must remain at the preceding value. It is not difficult to derive that for the k-th station the first column of the matrix contains the number

$$N_k - \max\{0, 1 - N_k^0 + N_1^0\}. \tag{13}$$

Example: We will again refer to Table V, where we can see that for station No 5 the number remains constant for five columns ($N_5^0 - N_1^0 = 7 - 2 = 5$). Station No 2 does not start from its maximum N_2 but, in keeping with (13), from the number $N_2 - \max\{0, N_1^0 - N_2^0 + 1\} = N_2 - 1 = 3$.

C. Optimized Algorithm

Based on Sections VI-A and VI-B, we introduce the optimized algorithm of computing the probability in point.

Algorithm 3 (Optimized computation of P_{win} for $K \geq 2$): Let the parameters N_k^0, N_k be given for all $k = 1, \ldots, K$.

1) (optional step) Find $m = \min_k\{N_k^0\}$ and replace N_k^0 by $(N_k^0 - m)$ for all k.
2) Compute $\Sigma_{k^*} = \min\{\Sigma_2, \ldots, \Sigma_K\}$.
3) Compute the number of significant columns:
   if $\Sigma_1 < \Sigma_{k^*}$ then $n_{\text{col}} = N_1$,
   else $n_{\text{col}} = \max\{0, \Sigma_{k^*} - N_1^0 - 1\}$ according to (10).
4) If $n_{\text{col}} = 0$ then $P_{\text{win}} = 0$ finish the Algorithm.
5) Allocate memory for a matrix of size $K - 1 \times n_{\text{col}}$.
6) Compute $\Sigma_k = N_k^0 + N_k$ for all k.
7) Fill the first column of the matrix: in the $(k-1)$-th row there will be $[N_k - \max\{0, N_1^0 - N_k^0 + 1\}]$.
8) If $n_{\text{col}} > 1$:
   Create the indicator vector ict of length $K - 1$ with value $[N_k^0 - N_1^0 - 1]$ in the $(k-1)$-th position.
   For $n = 2 : n_{\text{col}}$, fill in the column n in the matrix with column $n - 1$ reduced by 1 in positions where the logical expression $ict \leq 0$ holds true.
   Then, replace ict by $[ict - 1]$.
9) After filling the matrix, multiply all its elements in each column and then sum the resulting n_{col} numbers. Dividing this by $\prod_k N_k$, obtain P_{win}.

D. Further Optimization

Ideas for other improvements are straightforward. In practice, situations often occur that there are groups of stations sharing the same parameters. If such a case arises, one can replace the calculation of all the rows of the matrix in Algorithm 3 just by copying the corresponding row, computed only once, for all the stations belonging to the same category. Furthermore, the resultant probability of winning needs to be computed also only once within a category, the other stations from the respective group will gain the same value, see results in Tables III and IV.

VII. Software

The results established in the paper were empirically verified by a simulation run in Matlab (R2010a, 7.10.0.499).

Corresponding Matlab files can be downloaded from: http://www.mathworks.com/matlabcentral/fileexchange/27573 The demo script 'demo_802_11e_contention_edca.m' shows the computations corresponding to the two example scenarios considered (Table III and Table IV) and it utilizes the functions 'edca_probability_win.m' (corresponding to Algorithms 1 or 3) and 'edca_probability_collision.m' (corresponding to Algorithm 2). The other functions present are subsidiary.

VIII. Conclusion

The paper introduces an enhanced probabilistic algorithm allowing technicians to evaluate the effect of different WLAN MAC parameters on the probability of gaining access to the shared wireless communication medium for the corresponding 802.11e access categories. The method suggested was evaluated in a demonstration scenario and the results obtained match with our empirical expectations. The interpretation of these results was also given. On the other hand, we are aware that the model is still quite abstract and does not respect all important characteristics of real network traffic. For example, we did not deal with interrupted attempts to access the medium, with reactions to collisions and we did not consider variable traffic intensity for different access categories.

Appendix

We will prove the validity of inequality (11) under specified constraints imposed on i and j. We choose a fixed $k \in \{2, \ldots, K\}$.

Admissible i must comply with

$$\begin{aligned} 1 \leq i &\leq \Sigma_1 - N_1^0 \\ i &\leq N_k - N_1^0, \end{aligned}$$

which means, among other things, that $N_1^0 + i \leq \min\{\Sigma_1, N_k^0\}$. Then, of course, it holds $P(X_k > N_1^0 + i) = 1$, because $P(X_k > N_1^0 + i) \geq P(X_k > \min\{\Sigma_1, N_k^0\}) = 1$. These are in fact cases with such a value of i that the first station cannot lose the contention with the k-th station.

The case of the variable j is related to a situation when there is a non-zero chance that station k will win over the first station. Like above, the admissible j must satisfy the inequality $\max\{N_1^0, N_k^0\} < N_1^0 + j$ and therefore it holds
$P(X_k > N_1^0 + j) < P(X_k > \max\{N_1^0, N_k^0\}) = 1$.

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References

[1] Bianchi, G. Performance Analysis of the IEEE 802.11 Distributed Coordination Function, IEEE Journal on Selected Areas in Communications, Volume 18, Issue 3, Mar 2000, Pp. 535–547

[2] IEEE Std 802.11b-1999 (R2003), IEEE Standard for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Higher-Speed Physical Layer Extension in the 2.4 GHz Band, 1999, (reaffirmation 2003)

[3] IEEE Std 802.11a-1999 (R2003), IEEE Standard for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: High-speed Physical Layer in the 5 GHz Band, 1999, (reaffirmation 2003)

[4] IEEE Std 802.11g-2003, IEEE Standard for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Amendment 4: Further Higher Data Rate Extension in the 2.4 GHz Band, 2003

[5] IEEE Std 802.11-2007, IEEE Standard for Information technology – Telecommunications and information exchange between systems – Local and metropolitan area networks – Specific requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, USA, 2007

[6] Rajmic, P., Komosný, D. and Molnár, K., Theoretical Analysis of EDCA Medium Access-Control Method in Simplified Network Environment, In *Proceedings of the 8th International Conference on Networks, ICN 2009*, Gosier, Guadeloupe, IEEE Computer Society, 2009, ISBN 978-0-7695-3552-4

[7] Rajmic, P. and Molnár, K., Analytical Evaluation of IEEE 802.11e QoS Enhancement for Wireless Local Area Networks, In *Proceedings of the 32nd International Conference on Telecommunications and Signal Processing, TSP 2009*, Dunakiliti, Hungary, 2009, ISBN 978-963-06-7716-5